Markov Chains

Guillaume Obozinski

Swiss Data Science Center
EPFL & ETH Zürich

RLSS, juillet 2019

Independence concepts

Independence: $X \perp\!\!\!\perp Y$

We say that X et Y are independents and write $X \perp\!\!\!\perp Y$ ssi:

$$\forall x, y, \qquad P(X = x, Y = y) = P(X = x)\, P(Y = y)$$

Conditional Independence: $X \perp\!\!\!\perp Y \mid Z$

- On says that X and Y are independent conditionally on Z and
- write $X \perp\!\!\!\perp Y \mid Z$ iff:

$\forall x, y, z,$

$$P(X = x, Y = y \mid Z = z) = P(X = x | Z = z)\, P(Y = y | Z = z)$$

Review of Markov chains

Definition (Markov chain)

$\forall t \geq 0,\ \forall (x, y) \in \mathcal{X}$, a *state space*

$$p(X_{t+1} = y \mid X_t = x, X_{t-1}, \ldots, X_0) \qquad = p(X_{t+1} = y \mid X_t = x)$$

- Factorization: $p(x_1, \ldots, x_M) = p(x_1) \prod_{t=2}^{M} p(x_t | x_{t-1})$
- Graphical model: 

- The future is independent of the past given the present

Definition (Time Homogenous Markov chain)

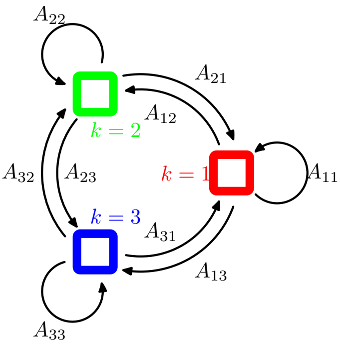
$$p(X_{t+1} = y \mid X_t = x) = p(X_1 = y \mid X_0 = x)$$

Transitions

Definition (Transition matrix)

If $\mathcal{X}$ is a finite set, A is a matrix with

$$A_{x,y} = A(x,y) = \mathbb{P}(X_t = y | X_{t-1} = x).$$



$$A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

$$A\mathbf{1} = \mathbf{1}$$

$$Au = \mathbb{E}[u(X_{t+1}|X_t = \cdot)]$$